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POSITION OF MC MURDO SOUND, ANTARCTICA TRACKING STATION AS DETERMINED BY ARTIFICIAL EARTH SATELLITES

During the Antarctic summer (1964-1965) a doppler tracking station, under cognizance of the United States Antarctic Research Program, National Science Foundation, was installed at McMurdo Sound, Antarctica with equipment furnished by the U. S. Navy. Initial station coordinates were acquired by taking an average value from various sources. Since there are no known triangulation or Hiran ties from Antarctica to any existing datum in the world, it was expected that the coordinates were probably in error with respect to the APL Reference System (Heuring, 1964). This was soon verified after some passes of data were processed by our tracking programs. Herein will be described the method and results of refining the position of the McMurdo Sound Doppler Tracking Site (hereinafter referred to as Station 019).

Method

Prior to defining the details of the calculation a brief foundation will be laid. Passes of doppler data are received from stations in the TRANET doppler tracking system (for an overall discussion of the tracking system see Newton, 1963). These data are processed and compared to similar data generated by an updated orbit, i.e., one derived from integrating the equations of motion to some particular time, t, thereby defining the satellite position, $\vec{r}(t)$, and velocity $\dot{\vec{r}}(t)$. Then the difference between the experimental and generated data is minimized by adjusting the initial conditions, i.e., the orbit parameters. At the minimum, the orbit is deemed an improved orbit.

These calculations are described in the orbital improvement programs (see Holland and Yingling [1963], Black [1963a], Welty [1963a], Welty [1963b], Datatrol Corporation [1963], Holland [1963], Albright [1963] and Monahan [1964]). Using the improved orbit, an assessment of the individual passes used to improve the orbit is made on a routine basis. Further, passes which have not been used to improve the orbit may be assessed. The latter type of computer run is termed an Editor run (ED) whereas, the former is a tracking run (T). Both type of runs were employed in adjusting the position of Station 019. Some of the tabulated results which were used from these runs are:

(1) along-track, ϵ_{a_i};

(2) slant range error, ϵ_{ρ_i};

(3) satellite elevation, E_i;

(4) satellite azimuth clockwise from north, A_i.

All of these values are evaluated at the point of closest approach of the satellite to the station where the subscript i refers to a particular pass in a run.

Along-track error is the difference in position of the station between that defined by the given station coordinates and that defined by the doppler data recorded at the station. Its positive direction is that of the relative velocity of the satellite, $\dot{\vec{r}}$, and station, $\dot{\vec{r}}_T$, i.e., $(\dot{\vec{r}} - \dot{\vec{r}}_T)$ [Black, 1963b]. The slant range error is similarly defined except that the positive direction is from station to satellite [Black, 1963b].

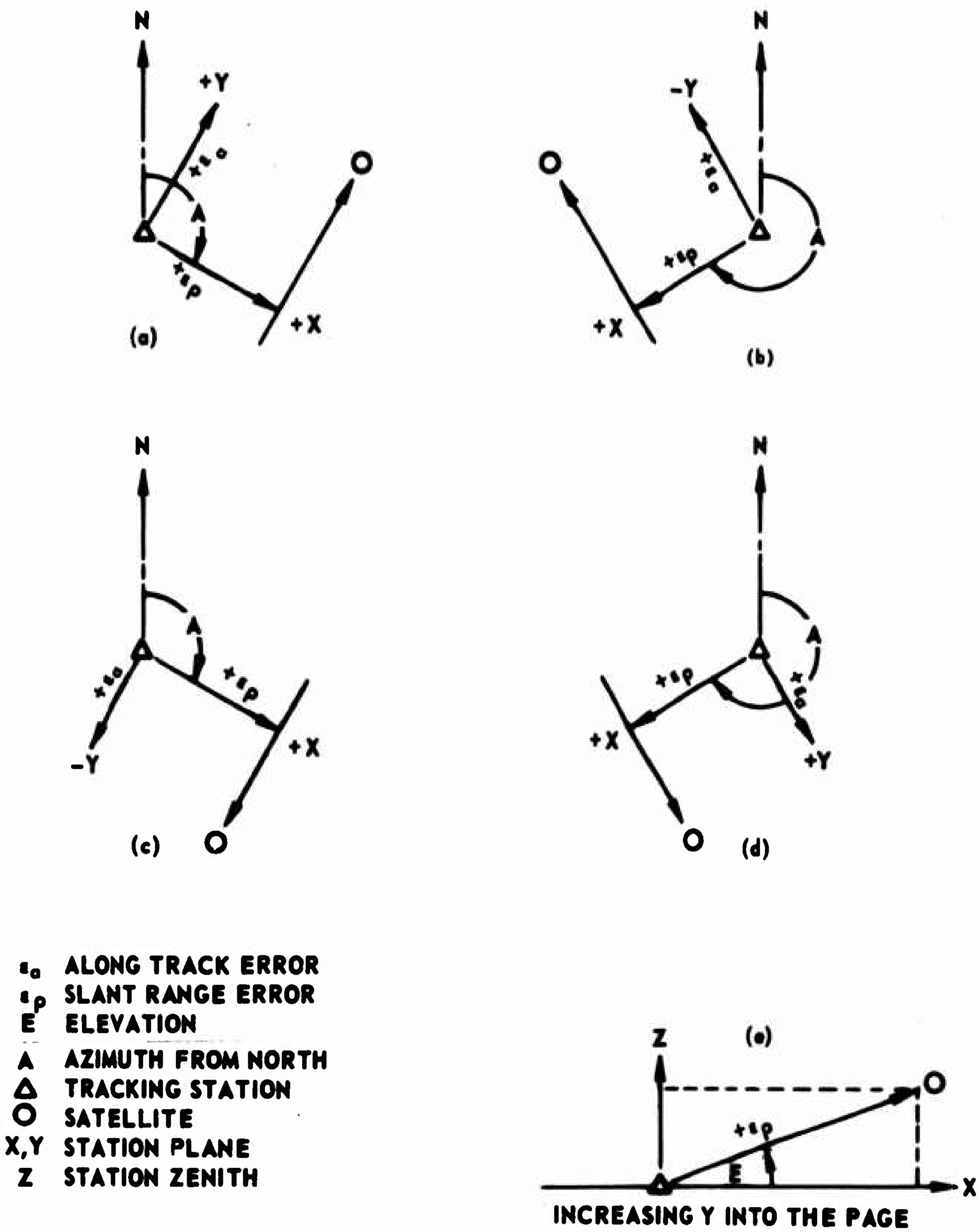
An orthogonal triad is defined by $\hat{\epsilon}_{\rho_1}$, $\hat{\epsilon}_{a_1}$ and $\hat{\epsilon}_{\rho_1} \times \hat{\epsilon}_{a_1}$, but the magnitude of the latter is not evaluated because of a very ill-conditioned matrix [Black 1963b]. Further details of this and other coordinate systems and inter-relations may be found in Guier [1963].

Contained in Figure 1 are four possible geometric configurations for a pass of a satellite where, at closest approach, representation (a) is a satellite moving northward to the east of a station, (b) is a satellite moving northward to the west of a station, (c) is similar to (a) but moving southward and (d) is similar to (b) but moving southward. Construct a right-handed cartesian coordinate system whose X and Y axes are in the station plane, with +Z in the station vector direction out of the earth and +X in the direction of the projection of the positive slant range vector onto the station plane (Figure 1e). In this X, Y, Z coordinate system we will express the displacement of the position of the station from its initial position according to ϵ_{a_1}, ϵ_{ρ_1}, A_1 and E_1. First, determine X_1, Y_1, Z_1.

$$X_1 = \epsilon_{\rho_1} \cos E_1$$

$$Z_1 = \epsilon_{\rho_1} \sin E_1$$

Because $0 \le E_1 \le \pi/2$, the sign of X_1 and Z_1 is governed by the sign of ϵ_{ρ_1}. By definition ϵ_{a_1} lies along the Y axis and in Figure 1 (a,d) + ϵ_{a_1} is in



FOR (b) AND (c), ε_a IS SET EQUAL TO $-\varepsilon_a$ BEFORE COMPUTATIONS BEGIN, SEE TEXT.

Fig. 1 GEOMETRIC CONFIGURATIONS FOR A SATELLITE PASS

the + Y direction, whereas in Figure 1 (b,c) + ϵ_{a_i} is in the -Y direction. For the latter cases, i.e., Figure 1 (b,c), set $\epsilon_{a_i} = - \epsilon_{a_i}$ so that all $\epsilon_{a_i} > 0$ are in the +Y direction. The adjusted station position is X_i, Y_i, Z_i. It remains to express X_i, Y_i, Z_i in the APL Reference System, i.e., X_A, Y_A, Z_A. The latter is here accomplished by three rotations and a translation. First rotate the X, Y, Z coordinates about the Z axis by $\psi_i = A_i - \pi/2$ so as to express the new position in a coordinate system X', Y', Z' with X' toward the east and Y' toward the north, i.e.,

$$\begin{pmatrix} X_i' \\ Y_i' \\ Z_i' \end{pmatrix} = \begin{pmatrix} \cos\psi_i & \sin\psi_i & 0 \\ -\sin\psi_i & \cos\psi_i & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} \qquad i = 1, \ldots, I$$

where I is the number of passes used to adjust the station position. Rotate about the X' axis such that the Z'' axis is parallel to the Z_A axis, i.e., $\zeta = \varphi_T - \pi/2$ where φ_T is the station spherical latitude used in ED.

$$\begin{pmatrix} X_i'' \\ Y_i'' \\ Z_i'' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\zeta & \sin\zeta \\ 0 & -\sin\zeta & \cos\zeta \end{pmatrix} \begin{pmatrix} X_i' \\ Y_i' \\ Z_i' \end{pmatrix} \qquad i = 1, \ldots, I$$

Rotate about the Z'' axis such that the X''' and Y''' axes are parallel to X_A and Y_A axes, i.e., $\delta = -(\lambda_T + \pi/2)$ where λ_T is the station spherical longitude used in ED.

$$\begin{pmatrix} X_i''' \\ Y_i''' \\ Z_i''' \end{pmatrix} = \begin{pmatrix} \cos\delta & \sin\delta & 0 \\ -\sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_i'' \\ Y_i'' \\ Z_i'' \end{pmatrix} \qquad i = 1,\dots, I$$

Finally, translate to X_A, Y_A, Z_A by the station cartesian coordinates x_T, y_T, z_T which were used in ED

$$\begin{pmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{pmatrix} = \begin{pmatrix} x_T \\ y_T \\ z_T \end{pmatrix} + \begin{pmatrix} X_i''' \\ Y_i''' \\ Z_i''' \end{pmatrix} \qquad i = 1,\dots, I$$

Form the mean values of the I sets of coordinates

$$\bar{x} = \sum_{i=1}^{I} \frac{X_{A_i}}{I}, \text{ etc.} \tag{1}$$

The new spherical coordinates are

$$\varphi = \tan^{-1} \frac{\bar{z}}{[\bar{x}^2 + \bar{y}^2]^{\frac{1}{2}}}$$

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$$\lambda = \tan^{-1} [\bar{y}/\bar{x}]$$

$$r = [\bar{x}^2 + \bar{y}^2 + \bar{z}^2]^{\frac{1}{2}}$$

With these coordinates an ED is made to test the validity of the station adjustment, i.e., determine if the rms total error (ϵ_{ρ_i} and ϵ_{a_i}) of the ED has been reduced. This process is continued until the rms total error is less than that of the orbit.

Results

Tracked orbits (day 043 and day 052) of satellite 03164, defined with no Station 019 data, were used with the Editor programs yielding slant range and along-track errors which allowed for adjustment of the Station 019 coordinates. Table 1 contains the step-by-step results where data span indicates the period in which the data used from Station 019 were observed; σ_T is the rms error of the tracked orbit; σ_{ED} is the rms error of the Station 019 data from an Editor run; Station 019 passes refers to the number of passes used in an Editor run; ED coordinates are those used in a particular Editor run; and, the adjusted coordinates are the result of correcting the ED coordinates via the Editor run results, i.e., ϵ_{ρ_i} ϵ_{a_i} etc. The adjusted coordinates become the ED coordinates for the next Editor run if, for a particular run, $\sigma_{ED} > \sigma_T$. If $\sigma_{ED} < \sigma_T$, further Editor runs with the same orbit are meaningless, for the station position error is less than the orbit error. This is the situation after runs four and six, and since the specified σ_T are typical values for standard tracking

runs the adjusted coordinates of run six would be deemed the best coordinates for Station 019.

The validity of these coordinates was assessed by tracking satellite 63041 on day 052 - 1965 with nine passes of Station 019 data and 64 passes of data from Stations 003, 008, 011, 012, 013, 014, 018, 106, 111, and 115 whose coordinates are well defined in the APL Reference System (see <u>Guier</u> [1965]).

Table 2 contains the tracking results using the run six adjusted coordinates for Station 019. The σ_T for all passes used in the tracking is greater than the σ_T for the Station 019 passes; thus, the adjusted coordinates (run six Table 1) are considered a good representation of the Station 019 position.

Using the $\bar{x}$, $\bar{y}$, $\bar{z}$ values (Eq. 1) for the adjusted coordinates of run six in Table 1 an approximation can be made for the geodetic coordinates φ_G, λ_G in the APL Reference System (<u>Heuring</u> [1963]) and appears in Table 3 along with the initial station coordinates. This is an approximate value because the distance $\bar{h}$ from $\bar{x}$, $\bar{y}$, $\bar{z}$ to the APL Ellipsoid ($a = 6378.166$ km and $f = 1/298.3$) includes the geoidal height as well as the station elevation above mean sea level (ostensibly the geoid) with only the latter known. However, the station elevation (35.6 m.) and distance $\bar{h}$ (~ -33.5 m, implying the ellipsoid is above the point $\bar{x}$, $\bar{y}$, $\bar{z}$) yield a geoidal height value, H, of about -70 m which is close to the geoidal height value of -50 in <u>Guier</u> [1965]. In view of the small values h, $\bar{h}$ and H the error incurred in specifying φ_G, λ_G from $\bar{x}$, $\bar{y}$, $\bar{z}$ is of the order of a meter or less. It is not

TABLE 1

RESULTS OF EDITOR RUNS USING TRACKED ORBITS OF SATELLITE 03164										
RUN NO.	DATA SPAN	σ_T (n.m.)	σ_{ED} (n.m.)	STA 019 PASSES	ED COORDINATES			ADJUSTED COORDINATES		
					φ	λ	r	φ	λ	r
1	043,044–1965	0.0456	0.314	18	−1.3574012	2.9087361	.99679498	−1.3573299	2.9089328	.99677287
2	043,044–1965	0.0456	0.077	18	−1.3573299	2.9089328	.99677287	−1.3573275	2.9089938	.99678075
3	043,044–1965	0.0456	0.052	18	−1.3573275	2.9089938	.99678075	−1.3573212	2.9090043	.99678315
4	043,044–1965	0.0456	0.043	18	−1.3573212	2.9090043	.99678315	−1.3573191	2.9090063	.99678585
5	052,053–1965	0.0409	0.0412	7	−1.3573191	2.9090063	.99678585	−1.3573197	2.9090022	.99679021
6	052,053–1965	0.0409	0.037	7	−1.3573197	2.9090022	.99679021	−1.3573182	2.9090045	.99679216

COORDINATES ARE SPHERICAL GEOCENTRIC WITH φ IN RADIANS > 0 IS NORTH, λ IN RADIANS > 0 EAST AND r IS IN UNITS OF 6,378.166 KM

TABLE 2

RESULTS OF TRACKING SATELLITE 63041	
DATA SPAN	DAY 052-1965
σ_T ALL PASSES (73)	0.0366
σ_T STATION 019 PASSES (9)	0.0313

TABLE 3

INITIAL AND FINAL GEODETIC COORDINATES

	φ	λ
INITIAL	77° 51' 10"S	166° 39' 30"E
FINAL	77° 50' 52.9"S	166° 40' 25.2"E

meant to imply that the position is known to a meter only that the transformation from spherical to geodetic coordinates is specified with this accuracy. The accuracy of the position depends on the orbit accuracy which in this case is less than 70 meters rms-wise.

This report is considered preliminary being based on a small data sample sent to us by the last possible mail prior to their 1965 winter. Accordingly, a more thorough study is contemplated at a later date.

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